

Heisenberg's Uncertainty Principle

Heisenberg's Uncertainty principle defines one of the many truths behind modern science that the average person is unaware of. In the frame of reference that we all exist in, objects have a location, as we observe; they are in the place we see them at and move within a given area of space. Newton described this intuitive framework as the bucket of the universe; things were within it, acted within it, and existed along a timeline that flows around it. Around the turn of the twentieth century, this all changed. A revolution in physics began, and everything needed to be redefined. Atomic particles, and consequentially the universe, behaved differently. We discovered that the smallest components of matter exist in a realm of possibilities. Instead of being here or there, they are both and other places, some areas more than others (at least more probably). During the act of observation, they escape from this other dimension and into a more familiar one, (this will later be described as a 'wave function collapse')... but they do not become completely defined. The limits of our observations (more specifically the limits of conjugate non-communicating attributes of particles such as speed and location, spin and axis of rotation, or energy and time) are defined by the set of equations and subsequent theories called Heisenberg's Uncertainty Principle.

Sir Isaac Newton surmised one of the first theories of the universe. Several of his conjectures were instinctive and came from normal observations. His theory is basically his famous three laws of motion. His description had only a few discrepancies, but the one that shattered our belief in this simple reality and the one we are primarily concerned with is the double slit test. First, allow me to mention a characteristic of a wave function,

like electromagnetic radiation, that a solid, or subatomic particle, does not share: diffraction. This is the attribute of a wave that sends it about an angle when it encounters an edge of a surface. This is no different than watching the ripples in water disperse in a fountain. Now when electrons (or atoms, small molecules, or any other atomic particles) are showered through a barrier with two adjacent openings, they pass through the openings and interact with one another on the other side as if they were no different than light, or a water surface. Waves possess another unique characteristic when they interact by reinforcing each other at peaks and cancelling each other out when a peak meets a trough. In other words, waves can add up or subtract each other when they come into contact. The concentration of electrons hitting a second barrier forms what is called an 'interference pattern' which shows that they behave in this manner. Someone might speculate that because there are so many electrons, they act like molecules in water. However, if a single electron is shot one at a time, several million times, the same pattern will appear showing that a single electron interacts with itself proving that it is in more than one place and in reality all possible places at the same time. Another experiment depicts this when a small hole is poked through a barrier and on the other side a target pattern is created. These wave-like properties seen in particles have subsequently been called the wave-particle duality of matter (stemming from the wave-particle duality of light when photons, particles of light, were proven to act with wave attributes similarly).

“Although we will concentrate on the wave-particle duality of electrons because it is important in chemistry and the physics of atoms, all the other “particles” of matter you've learned about show wave properties as well...

◇ What is the wavelength of a trotting elephant?

◇ One may doubt whether the equation should be applied to an elephant, which is not just a single particle but a rather large collection of them. Throwing caution to the wind, however, we estimate the elephant's mass at 10^3 kg and its trotting speed at 10 m/s. Its wavelength is therefore roughly

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(10^3 \text{ kg})(10 \text{ m/s})} \\ &\sim 10^{-37} \frac{\{\text{kg}\cdot\text{m}^2/\text{s}^2\}\cdot\text{s}}{\text{kg}\cdot\text{m/s}} \\ &= 10^{-37} \text{ m} . \end{aligned}$$

The wavelength found in this example is so fantastically small that we can be sure we will never observe any measurable wave phenomena with elephants. The result is numerically small because Planck's constant is so small.” (Crowell Example 1).

So, matter exists in a wave-structured probability state prior to being measured. This statement sets the scene for Heisenberg to discover that as particles lack position and momentum before observation, they can never have a definitive position and momentum. And this doesn't come from an inability for humans to look closely enough, or to be misunderstood with the observer effect (in the act of observing, even minutely, we affect the system. Even taking a video of an electron, by reflecting the light off of it, the action disturbs its natural state). There is just limited information to be taken from these particles. The maximum information attainable from two attributes in the same category, called non-communicating attributes, can be found using Heisenberg's Uncertainty Principle.

Werner Karl Heisenberg was born December 5th 1901 in Wurzburg Germany. He excelled in mathematics at an early age, and piano as well (he enjoyed playing throughout his life). He taught himself calculus and was accepted into the University of Munich to study physics, despite his desire to study pure mathematics. He received his doctorate in 1923, went into theoretical physics, and entered the field which recently had undergone a siege of new and controversial findings. Heisenberg studied with Max Born and with Niels Bohr at Copenhagen. After an era of civil unrest due to World War I, Heisenberg published his first paper on the “Zeeman effect” and became well known for it. Over the next few years, he developed the matrix form of quantum mechanics; Heisenberg, Born, and an assistant Pascual Jordan created the foundation for matrix mechanics. However, their work was overshadowed by Schrödinger’s wave mechanics. While studying transformation theory, Heisenberg formulated the uncertainty principle and declared that two conjugate attributes will, in principle, never be simultaneously known. This discovery reverberated in the scientific community for decades. Bohr, working with Heisenberg, contributed the principle of complementarity (a description of the wave attributes and particle attributes that describes them as complementing each other as opposed to contradicting each other) , and both interpreted Schrödinger’s wave function as probability density. These three principles laid the foundation for the widely accepted “Copenhagen Interpretation”. Despite its popularity, a very select few were unsatisfied with it, including Einstein, Schrödinger, and Planck. Heisenberg went on to teach in Leipzig as Germany’s youngest professor. He later formulated a unification of quantum mechanics and relativity, although it left discrepancies. Another triumph of Heisenberg’s was his model of an atomic nucleus that contained both protons and

neutrons. In late 1933, he was awarded the noble prize for his contributions to quantum mechanics. Another era of political unrest led to the stagnation of Heisenberg's pursuits when he was to succeed Sommerfeld's position, but met with opposition from Nazi physicists. He went on to study high and low energy interactions of particles. History repeated itself when Heisenberg was offered the same position, but a smear campaign came from the S.S. He could never take the position. World War II came and challenged Heisenberg's loyalty to Germany. While his personal safety was compromised, he chose to remain there and continue research. Towards the end of the war, Heisenberg among other physicists were taken from Germany and eventually turned over to the British. After being returned to Germany at the end of the war, Heisenberg headed a team of scientists as advisors to the chancellor of Germany. He fought through more political turbulence regarding nuclear empowerment over the next years and later picked up where he left off in formulating a consistent quantum field theory. With the help of Pauli, he developed a new system of equations that described the behavior of every elementary particle. Called the non-linear spinor theory, it seemed to satisfy Heisenberg on many levels. He died in December 1975.

Although Heisenberg by no means discovered this principle by himself, his contributions were the most influential. Another simple scenario to outline the equation of the Uncertainty Principle is to imagine an atomic particle in some bounded region, the possible positions of this particle range from one end off the region to the other: the length L . Another way to define its position is to label the realm of where is could possibly be Δx , however, we are still uncertain of its specific location. We only know that it is within this domain, so consequentially this is measure of our uncertainty.

Likewise Δp is the realm of momentum that we are able to measure. The following quote gives the mathematical proofs of the relation found between these two variables.

“[These] discussions are based on the particle-wave duality as expressed by the Einstein-de Broglie relations... [Position is equal to plank’s constant divided by the wave function.]

... Consider next the following thought-experiment of measuring the position of a particle by means of a microscope. Now the accuracy of position is limited by diffraction effects which cause the image to blur. The resolving power, namely, the smallest separation Δx between two points whose images are resolved, is [the wave function divided by the Sine of the angle between them.]

Where the wave function is the wavelength of the illuminating light, and the angle is the aperture angle of the objective lens of the microscope...

Now it is known from the Compton Effect that when light is scattered by the particle into the objective of the microscope, the particle receives recoil. But as the scattered photon may enter the objective in any direction within the aperture [of two times the angle], the corresponding recoiled particle may have a recoil velocity component over a corresponding range, as determined by the energy-momentum conservation laws in this Compton scattering. It can be seen that corresponding to the angular spread of the scattered photon, the spread of the x-component of the recoil momentum is plank’s constant over the wave function multiplied by the sin of the angle.

Thus [when multiplied, the wave function and Sine cancel out] one has

$$\Delta x \Delta p = h.$$

Consider another experiment in which a beam of particles is incident perpendicularly on a screen with a slit of width Δx . Thus the indeterminacy of the particle in the x-direction is Δx . Now from the wave point of view, a wave of wavelength passing through a slit of width Δx will undergo diffraction, resulting in a deflection of angle given by: sine is equal to the wave function divided by the width Δx .

According to the de Broglie relation momentum is plank's constant divided by wave function, the x-component of the momentum is determined using this and the sine of the angle. Due diffraction the [particle may be deflected by either by a positive or negative angle, so that the uncertainty in the x-component of the beam (after passing through the slit) is plank's constant times sine of the angle over the wave function.

From [this], one has again

$$\Delta x \Delta p = h.$$

It is important to note that, while the interactions between the object of a measurement and the measuring devices cause finite but non-vanishing corrections for them to be made. This in principle can be done [in] classical physics. The point is that, in quantum mechanics corrections for the disturbances arising from the instruments *cannot* be made to accuracy greater than that allowed by the constant h . To repeat, the effect of a measuring process disturbing the object of measurement, by itself, will not lead to the uncertainty relation; it is the limitation, set by the constant h through the Einstein- de Broglie relations, on the accuracy of making the corrections that's in point. In classical physics, $h = 0$ and in *principle*, it is possible to make exact corrections and there is no "uncertainty relation". Einstein later attempted to refute this with more thought

experiments, but in each case, an application of this process was shown by Bohr and Einstein unhappily agreed. (Ta-You Wu in “Published Lectures on Quantum Mechanics”).

Heisenberg is credited for taking these formulas and translating them in reality; he showed the physical results of these equations. As the particles are in the process of traversing around these obstacles, a wave function represents their possible locations. The probability of a certain location is the amplitude (height) of the wave squared. Put quaintly, ‘probability equals possibility squared’. Whenever we take a measurement, look to find where the particle is located, the wave function we have been describing collapses to the single location where we observe. How this happens is a controversial difference between many universal theories among the Copenhagen Theory, Many Worlds Theory, and Timeless Theory (specifically that defined by Julian Barbour, some theories incorporate the possibility of timelessness due to time’s absence in quantum equations). The most widely accepted theory is the Copenhagen Theory, but a close second is most likely the Many Worlds. The Many Worlds interpretation states that for each possible location among the wave function, a separate universe exists for that possibility. What seems as a wave function collapse is the distinction between the universes and the path we take when we choose one universe to exist along. In the Copenhagen Interpretation, the wave function or, “the probabilistic statements reflect a fundamental property of nature, not simply our ignorance. It is not that before the measurement the particle does have a definite momentum and we simply do not know it. Instead, all momenta in the superposition are present as potentialities, and measurement forces one of them to be actualized. This is justified by a simple and persuasive fact. If we do not perform

measurement but allow to evolve, and only later make some measurement, then the things observed latter... are impossible le to explain unless all states were present initially and throughout the subsequent evolution”. (Julian Barbour in “The End of Time”).

Both the Uncertainty Principle and its founder, Werner Heisenberg, had profound influences in quantum physics. The Copenhagen Interpretation, which some might argue mostly cam from Heisenberg, is still in use today by quantum physicists to explain the nature of our universe. Scientists and civilians alike wonder if the universe will ever be completely understood or defined mathematically. We were close once in Newton’s time, it seemed. But we also thought we were close when we understood the world to be flat. Maybe some things will be refuted and redefined, but for the most part, we are sure of everything explained in this text. As far as the interpretations or what is further to be discovered, anything is possible; it is left up to future generations. The world is waiting for a very significant expanse in the frontier of human understanding in the soon to be world’s largest experiment. Within the next year, the Large Hadron Collider is set to fire and two protons traveling at 99.9% the speed of light will smash together and hopefully will reveal a gross of practical information to be used in quantum mechanics, quantum physics, and universal interpretations.

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